Semester Courses of M.A/M.Sc. (Mathematics)

The course of M.A/M.Sc. (Mathematics) will be spread in two years - Previous & Final. There will be four semester examinations and a viva-voce & project work examination in each semester.

M.A./M.Sc. Previous (Mathematics)

(Effective from session 2017-2018)

The M.A./M.Sc. Previous (Mathematics) examination will consist of two semesters, called first and second semesters. Their examinations will be held in the months of December and May respectively. In each of these semester examinations there will be five compulsory papers. Each paper will be of three hours' duration and of 50 maximum marks, except where stated otherwise. There will be a viva-voce and project work examination of 50 marks in each semester.

Format of the Question Paper:-

There will be one compulsory question consisting of 5 parts of short answer type question based on the whole course, out of which all parts will have to be answered. Besides, there will be 6 questions in two sections, ordinarily consisting of two parts (a) and (b), out of which 4 questions will have to be answered. Thus in all 5 questions will have to be attempted and 7 questions will have to be set. All questions will carry equal marks, except where stated otherwise.

First Semester

Compulsory Papers

Paper I	:	Groups and Canonical Forms
Paper II	:	Topology
Paper III	:	Differential and Integral Equations
Paper IV	:	Complex Analysis
Paper V	:	Real Analysis

Second Semester

Compulsory Papers

Paper I	:	Fields and Modules
Paper II	:	Differential Geometry of Manifolds
Paper III	:	Partial Differential Equations
Paper IV	:	Operations Research
Paper V	:	Hydro Dynamics

Viva –Voce and Project Work: **50 marks**

There will be a Viva-Voce and Project Work examination of 50 marks in each semester based on all the papers of respective semester. Under the project, the candidate will present a dissertation in his/her own handwriting. The dissertation will consist of at least one theorem/article with proof and one problem with solution, relevant definitions with examples and/or counter-examples, wherever necessary, from each paper of Mathematics studied in First and Second semesters.

Distributions of marks are as follows:-

1.	Attendance (To be verified by Head in case of University and by Principal in case of college)	**05 marks**	a.91%-100%- 05marks, b.81%-90%- 04marks, c.75%-80%- 03marks
2.	Presentation of at least 15 minutes to be evaluated by internal examiner	**10 marks**	
2.	Class test in each paper	**10 marks**	Average of all five tests to be taken
3.	Dissertation	**05 marks**	
4.	Viva-Voce	**20 marks**	

For viva-voce examination and evaluation of project work there will be a board of examiners consisting of an external examiner and an internal examiner. The dissertation will be forwarded by the Head of Department at the university centre and by the Principal of the college at the college centre.

M.A./M.Sc. Semester-I

Paper I- Groups and Canonical Forms

Groups : Conjugacy relation. Normaliser of an element. Class equation of a finite group. Center of a group. Fundamental theorems on isomorphism of groups. Automorphisms. Inner automorphism. Maximal subgroups. Composition series. Jordan-Holder theorem. Solvable groups. Nilpotent groups. Commutator subgroups. External and iternal direct product of groups. Cauchy's theorem for finite group. Sylow's theorem.

(4 questions)

Canonical forms: Similarity of linear transformations. Invariant subspaces. Reduction to triangular forms .Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent transformation. The primary decomposition theorem. Jordan blocks and Jordan forms.

(2 questions)

Books Recommended:-

1. I.N. Herstein : Topics in Algebra, Wiley Eastern Ltd., New Delhi.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul : Basic Abstract Algebra (Second Edition), Cambridge University Press, Indian Edition.
3. Surjeet Singh and Qazi Zameeruddin: Modern Algebra, Vikas Publishing House. Pvt. Ltd.
4. K.B. Datta : Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi,.
5. S. Kumaresan : Linear Algebra, A Geometric Approach, Prentice Hall of India.
6. A.R. Vasishtha & A.K. Vasishtha : Modern Algebra, Krishna Prakashan Media (P) Ltd., Meerut .
7. H.K.Pathak: Abstract Algebra, Shiksha Sahitya Prakashan.

Paper II-Topology

Definition and examples of topological spaces. Closed sets. Closure. Dense subsets. Neighbourhoods. Interior, exterior and boundary. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology. Alternate methods of defining a topology in terms of Kuratowski Closure operator and Neighbourhood Systems.

Continuous functions and homeomorphism. First and second countable spaces. Lindelof's theorems. Separable spaces. Second Countability and Separability.

(3 questions)

Separation axioms T_0, T_1, T_2, T_3, T_4; their characterizations and basic properties. Urysohn Lemma. Tietz extention theorem.

Compact sets and their properties. Finite intersection property, Bolzano Weierstrass property. Continuous functions and compactness, Sequential compactness, countable compactness and their comparison. One point compactification. Connected spaces. Connectedness on the real line. Components. Locally connected Spaces.

(3 questions)

Books Recommended:-

1. George F. Simmons : Introduction to Topology and Modern Analysis, Mc Graw-Hill Book Company.
2. J.L. Kelley : General Topology, Van Nostrand, Reinhold Co., New York.
3. K.D. Joshi : Introduction to General Topology, Wiley Eastern Ltd.
4. James R Munkres : Topology, Prentice Hall of India Pvt. Ltd., New Delhi.
5. Willard : General Topology Addison-Wesley, Reading.

Paper III-Differential and Integral Equations

Solution of Differential Equations in ascending and descending power series, Hypergeometric Differential Equations, Papperitz symbol, Pochhammer symbol, Hypergeometric Function, Solution of Gauss's Hypergeometric Differential Equation, Differentiation of Hypergeometric Functions.

Legendre's Differential Equation, Legendre's Functions, Generating Function for P_n (x), Laplace Definite Integrals for P_n (x), Orthogonal Properties of Legendre's Polynomials, Recurrence Formulae, Beltrami Result, Christoffel's Expansion and Summation formulae, Rodrigue's Formula for P_n(x).

Bessel's Differential Equation, Bessel's Functions, Generating Function for J_n (x), Differential Equations Reducible to Bessel's Differential Equations, Orthogonality of Bessel's Functions.

(3 questions)

Integral Equation, Linear Integral Equations, Types of Linear Integral Equations, Types of Kernels, Conversion of differential equations to integral equations, L_2 kernels and L_2 Functions, Eigen values and eigen functions, Solution of Volterra Integral Equations by Successive Approximations and Successive Substitution Methods.

Fredholm Integral Equations of First and Second kinds, Solution of Fredholm Integral Equations by Successive Approximations and Successive Substitution Methods, Neumann Series, Volterra solution of Fredholm Integral Equation of second kind, Reduction of Volterra Integral Equation into differential equation, Reduction of Volterra Integral Equation of first kind into Volterra Integral Equation of second kind.

(3 questions)

Books Recommended:-

1. Differential and Integral Equations by B.P. Parashar
2. Series Solution and Special Functions by V. S. Verma
3. Fundamentals of Integral Equations by V. S. Verma

Paper IV-Complex Analysis

Analytic continuation. Uniqueness of analytic continuation. Power series method of analytic continuation. Branches of many-valued function. Singularities of an analytic function. Riemann surfaces. Gamma function. Zeta Function. Principle of reflection. Hadamard's multiplication theorem. Functions with natural boundaries.

(3 questions)

Maximum-modulus theorem. Schwarz's lemma. Vitali's convergence theorem. Hadamard's three-circles theorem. Mean values of |f(z)|. Borel-Cartheodory theorem. Pharagmen-Lindelof theorem.

Conformal representation. Linear (bilinear) transformations involving circles and half-planes. Transformations $w=z^2$, $w=(z+1/z)/2$, $w = \log z$, $w = \tan^2 (z/2)$ Simple function and its properties. The "1/4 theorem".

Radius of convergence of the power series. Analyticity of sum of power series. Position of the singularities.

(3 questions)

Books Recommended:-

1. E.C. Titchmarsh: Theory of Functions, Oxford University Press, London.
2. Mark J. Ablowitz and A.S. Fokas: Complex Variables: Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.
3. R.V. Churchill & J.W. Brown. Complex Variables and Applications, 5th Edition McGraw-Hill, New York, 1990.
4. Shanti Narayan: Theory of Functions of a Complex Variable, S. Chand & Co., New Delhi.
5. H.K.Pathak: Complex Analysis, Shiksha Sahitya Prakashan

Paper V-Real Analysis

Definition and Existence of Riemann- Stieltjes integrals. Properties of the integral, integration and differentiation, the fundamental theorem of calculus, integration of vector valued functions. Rectifiable curves. Rearrangements of terms of a series, Riemann's theorem. Sequences and series of functions of real numbers. Pointwise convergence and uniform convergence. Cauchy Criterion of uniform convergence. Weierstrass M- test, Abel's and Dirichlet's tests for uniform convergence. Uniform convergence and continuity. Uniform convergence and integration. Uniform convergence and differentiation. Weierstrass approximation theorem,Power series, uniqueness theorem for power series, Abel's and Tauber's theorem.

(4 questions)

Functions of several variables, linear transformations, Derivatives in an open subset of R^n, Chain rule , Partial derivatives, interchange of the order of differentiation, derivative of higher orders, Taylor's theorem, Inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method.

(2 questions)

Books Recommended:-

1. Walter Rudin: Principles of Mathematical Analysis (3^{rd} edition), McGraw-Hill, Kogakusha, 1976 International Student Edition.
2. H. L. Royden : Real Analysis, Macmillan Pub. Co. Inc. New York, 4^{th} Edition, 1993.
3. Richard Johnson Baugh: Foundation of Mathematical Analysis.
4. H.K.Pathak : Real Analysis, Shiksha Sahitya Prakashan

M.A./M.Sc. Semester-II

Paper I- Fields and Modules

Field theory: Extension fields. Algebraic and transcendental extensions. Splitting field. Separable and inseparable extensions. Normal extension. Perfect fields. Finite fields. Antomorphisms of extensions. Galois group. Fundamental theorem of Galois theory. Construction with ruler and compass. Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals.

(4 questions)

Modules : Cyclic modules. Simple modules . Semi-simple modules. Schuler's lemma. Free modules. Noetherian and artinian modules. Hilbert basis theorem.

(2 questions)

Books Recommended:

1. I.N. Herstein : Topics in Algebra, Wiley Eastern Ltd., New Delhi.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul : Basic Abstract Algebra (Second Edition), Cambridge University Press, Indian Edition.
3. Surjeet Singh and Qazi Zameeruddin: Modern Algebra, Vikas Publishing House. Pvt. Ltd.,.
4. K.B. Datta : Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi.
5. S. Kumaresan : Linear Algebra, A Geometric Approach, Prentice Hall of India.
6. A.R. Vasishtha & A.K. Vasishtha : Modern Algebra, Krishna Prakashan Media (P) Ltd., Meerut .
7. H.K.Pathak: Abstract Algebra, Shiksha Sahitya Prakashan.

Paper II- Differential Geometry of Manifolds

Definition and examples of differentiable manifold. Differentiable functions. Differentiable curves .Tangent space .Vector fields. Lie bracket. Invariant view point of connections . Covariant differentiation . Torsion. Curvature . Parallelism . Difference tensor of two connections. Lie derivative.

(3 questions)

Riemannian Manifold. Riemannian connection. Riemannian curvature tensor and Ricci tensor. Idenitities of Bianchi. Sectional curvature.

Exterior product of two vectors. Exterior algebra of order r .Exterior derivative .Cartans's structural equations. Submanifolds. Normals. Induced connection. Gauss formulae. Weingarten formulae. Lines of curvature. Mean curvature. Equations of Gauss and Codazzi.

(3 questions)

Books Recommended:

1. B.B. Sinha : An Introduction to Modern Differential Geometry, Kalyani Publishers, New Delhi,
2. N.J. Hickls : Notes on Differential Geometry.
3. K. Yano and M. Kon: Structure of Manifolds, World Scientific Publishing Co. Pvt. Ltd.,1984.

Paper III- Partial Differential Equations

Partial Differential Equations of the First Order : Origin of first order partial differential equations. Lagrange's solution of first order linear partial differential equation. Non-linear partial differential equations of the first order. Cauchy's method of characteristics, Charpit's method and Jacobi's method.

(3 questions)

Partial Differential Equations of Second and Higher Orders : Origin of second order partial differential equations. Higher order partial differential equations with constant coefficients. Equations with variable coefficients. Classification of second order partial differential equations. Canonical forms. Solution of non-linear second order partial differential equations by Monge's method. Method of separation of variables for solving Laplace, wave and diffusion equations.

(3 questions)

Books Recommended:

1. V.S.Verma : A Text Book of Partial Differential Equations
2. A.R. Forsyth : A Treatise on Differential Equations
3. I.N. Sneddon : Elements of Partial Differential Equations.

Paper IV- Operations Research

Origin and development of OR. Objective, nature, definition and scope of OR. Phases of OR Methods.

Network analysis: Basic concepts and definition. Network drawing and analysis Critical path method. Labelling method. Methods based on time estimates to find critical path. Concept of slack and float. Resource levelling and time-cost trade-off analysis. Time-cost optimization procedure. Project crashing. PERT. Requirements for application of PERT technique. Practical limitations in using PERT. Diffferences in PERT and CPM. Shortest path problem. Minimum spanning tree problem. Maximum flow problem. Minimum cost flow problem.

(3 questions)

Sequencing Problems : Assumptions for seqnencing problem. Processing n jobs on two machines, n jobs on three machines, 2 jobs on m machines.

Non-Linear Programming: Introduction and defintions. Formulation of non-Linear programming problems, General non-linear programming problems. Constrained optimization with equality constraints. Constrained optimization with inequality constaints. Saddle point problems Saddle points and NLPP.

Goal Programming: Introduction and definition. Concept of goal programming. Difference between linear programming approach and goal programming approach. Goal programming model formulation. Methods of solution of goal programming problem. Graphical method and Simplex method.

(3 questions)

Books Recommended:

1. H.A. Taha: Operations Research – An Introduction, Macmillan Publishing Co., Inc., New York.
2. Kanti Swarup, P.K. Gupta, Man Mohan: Operations Research, Sultan Chand and Sons, New Delhi.
3. B.S. Goel, S.K. Mittal: Operations Research, Pragati Prakashan, Meerut.
4. P.K. Gupta, D.S. Hira: Operatons Research – An Introduction, S. Chand & Company Ltd., New Delhi.
5. S.S. Rao: Optimization Theory and Applications, Wiley Eastern Ltd., New Delhi.
6. J.K. Sharma: Operations Research – Theory and Applications, Macmillan India Ltd.
7. S.D. Sharma: Operations Research, Kedar Nath Ram Nath & Company.

Paper V- Hydro Dynamics

Lagrangian and Eulerian methods. Equation of continuity. Boundary surfaces. Stream Lines. Path lines and streak lines. Velocity potential. Irrotational and rotational motions. Lagrange's and Euler's equations of motion. Pressure equation. Bernoulli's theorem. Impulsive actions. Flow and circulation. Permanence of irrotational motion. Stream function. Irrotational motion in two-dimensions. Complex velocity potential. Sources, sinks, doublets and their images.

(3 questions)

Two-dimensional irrotational motion produced by motion of circular and elliptic cylinders in a liquid. Kinetic energy of liquid. Milne-Thomson circle theorem. Theorem of Blasius. Stoke's stream function. Motion of a sphere through a liquid. Vortex motion. Vortex lines. Kelvin's proof of permanence. Motion due to circular and rectilinear vortices.

(3 questions)

Books Recommended:

1. B.G. Verma: Hydrodynamics, Pragati Prakashan, Meerut, 1995.
2. W.H. Besaint and A.S. Ramsey: A Treatise on Hydrodynamics, Part II, C.B.S. Publishers, Delhi, 1988.
3. F. Chorlton: Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.

M.A./ M.Sc. Final (Mathematics)

(Effective from session 2018-2019)

The M.A./M.Sc. Final (Mathematics) will consist of two semesters, called third and fourth semesters. Their examinations will be held in the months of December and May respectively. In each of these semester examinations there will be three compulsory papers and two optional papers selected from each group of optional papers. Each paper will be of three hours' duration and of 50 maximum marks, except where stated otherwise. There will be a viva-voce and project work examination of 50 marks in each semester.

Format of the Question Paper.

There will be one compulsory question consisting of 5 parts of short answer type question based on the whole course, out of which all parts will have to be answered. Besides, there will be 6 questions in two sections, ordinarily consisting of two parts (a) and (b), out of which 4 questions will have to be answered. Thus in all 5 questions will have to be attempted and 7 questions will have to be set. All questions will carry equal marks, except where stated otherwise.

Third Semester

Compulsory Papers

Paper I	:	Dynamics of Rigid Bodies
Paper II	:	Banach Spaces
Paper III	:	Fluid Dynamics

Optional Papers

Any one of the following will have to be opted from each group

Group-1

Paper IV (a)	:	Riemannian Geometry
Paper IV (b)	:	General Relativity and Gravitation
Paper IV (c)	:	Complex Manifolds
Paper IV (d)	:	Advanced Topology

Group-2

Paper V (a)	:	Discrete Mathematics
Paper V (b)	:	Mathematical Modelling
Paper V (c)	:	Numerical Solution of Differential Equations
Paper V (d)	:	Algorithm and Data Structure

Fourth Semester

Compulsory Papers

Paper I	:	Measure Theory
Paper II	:	Hilbert Spaces
Paper III	:	Analytical Dynamics

Optional Papers

Any one of the following will have to be opted from each group

Group-1

Paper IV (a)	:	Finsler Geometry
Paper IV (b)	:	Cosmology
Paper IV (c)	:	Contact Manifolds
Paper IV (d)	:	Wavelet Analysis

Group-2

Paper V (a)	:	Information Theory
Paper V (b)	:	Bio Mathematics
Paper V (c)	:	Magneto Hydrodynamics
Paper V (d)	:	Computational Mathematics

Viva –Voce and Project Work: 50 marks

There will be a Viva-Voce and Project Work of 50 marks in each semester examination based on all the papers of respective semester. Under the project, the candidate will present a dissertation in his/her own handwriting. The dissertation will consist of at least one theorem/article with proof and one problem with solution, relevant definitions with examples and/or counter-examples, wherever necessary, from each paper of Mathematics studied in third and fourth semesters.

Distribution of marks are as follows:-

1.	Attendance (To be verified by head In case of University and Principal In case of college)	**05**	a.91%-100%- 05marks, b.81%-90%- 04marks, c.75%-80%- 03marks
2.	Presentation of at least 15 minutes to be evaluated by internal examiner	**10**	
3	Class test in each paper	**10**	Average of all five tests to be taken
4.	Dissertation and Viva-Voce	**05**	
5.	Viva-Voce	**20**	

For viva-voce examination and evaluation of project work there will be a board of examiners consisting of an external examiner and an internal examiner. The dissertation will be forwarded by the Head of Department at the university centre and by the Principal of the college at the college centre.

M.A./M.Sc. Semester-III

Paper – I Dynamics of Rigid Bodies

Harmonic oscillators. Effect of disturbing force. Damped and forced oscillations. Motion of a rigid body in two dimensions under finite and impulsive forces. Kinetic energy and moment of momentum in two dimensions. Rolling spheres and cylinders. Conservation of energy and momentum.

(3 questions)

Motion of a billiard ball. Equations of motion and their applications in three dimensions. Motion of a system of particles. Moving axes. Equations of motion in most general form. Momentum of a rigid body. Euler's equations of motion. Moment of momentum about instantaneous axis. Kinetic energy of a rigid body. Motion relation to earth's surface.

(3 questions)

Books Recommended:-

1. S.L. Loney: An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies, Macmillan India Ltd., 1982.
2. A.S. Ramsey: Dynamics Part-II, The English Language Book Society and Cambridge University Press, 1972.
3. J.L. Synge and B.A. Griffith: Principles of Mechanics, McGraw Hill International Book Company, 1982.

Paper – II Banach Spaces

Normed linear spaces, Banach spaces, their examples including R^n ,C^n , l_p (n), $1 \leq p < \infty$, c0 ,c, l_p , $1 \leq p < \infty$, P [a,b], C[a,b]. Joint continuity of addition and scalar multiplication. Summable sequences and completeness. Subsapaces, Quotient spaces of normed linear space and its completeness.

Continuous and bounded linear operators and their basic properties. Normed linear space of bounded linear operators and its completeness. Various forms of and operator norm.

(3 questions)

Isometric isomorphism, Topological isomorphism. Equivalent norms. Finite dimensional normed spaces and compactness. Riesz Theorem, Bounded linear functionals Dual spaces. Form of dual spaces $(R^{n)}$*, (C^n)*,c_o^*, l_1^* , l_p^*, $1<p<\infty$.

Hahn- Banach theorem for real and complex normed linear spaces and its simple consequences. Open mapping theorem and its simple consequences. Product normed space. Closed graph theorem. Uniform boundedness. Banach-Steinhaus theorem. Embedding and Reflexivity

(3 questions)

Books Recommended:

1. P.K. Jain, O.P. Ahuja and K. Ahmad: Functional Analysis, New Age International (P) Ltd. and Wiley Eastern Ltd., New Delhi, 1997.
2. B. Choudhary and S. Nanda: Functional Analysis with Applications, Wiley Eastern Ltd., 1989.
3. I.J Maddox: Functional Analysis, Cambridge University Press (1970).
4. G.F. Simmons: Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, New York, 1963.
5. K. Chandrashekhara Rao. Functional Analysis, Narosa Publishing House, New Delhi

Paper III Fluid Dynamics

Wave motion in a gas. Speed of Sound. Equation of motion of a gas. Subsonic, sonic and supersonic flows of a gas. Isentropic gas flows. Flow through nozzle. Shock formation. Elementary analysis of normal and oblique shock waves. Derivation of speed of shock formed by sudden movement of piston in a gas at rest.

(3 questions)

Stress components in a real fluid. Relations between Cartesian components of stress. Rate of strain quadric. Principal stresses. Relations between stress and rate of strain. Coefficient of viscosity. Navier-Stokes equations of motion. Steady viscous flow between parallel planes and through tubes of uniform circular cross-sections. Steady flow between concentric rotating cylinders. Diffusion of vorticity. Energy dissipation due to viscosity. Reynolds number.

(3 questions)

Books Recommended:-

1. F. Chorlton: Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985

Paper IV (a) Riemannian Geometry

Hypersurfaces : Unit normal. Generalised convariant differentiation. Gauss's formulae. Curvature of a curve in a hypersurface. Normal curvature. Mean curvature. Principal normal curvature. Lines of curvature. Conjugate and asymptotic directions. Tensor derivative of the unit normal. Gauss characteristic equation and Mainardi-Codazzi equations. Totally geodesic hypersurfaces.

Subspaces: Unit normals. Gauss's formulae. Change from one set of normals to another. Curvature of a curve in subspace. Conjugate and asymptotic directions. Generalisation of Dupin's theorem. Derived vector of a unit normal. Lines of curvature for a given normal.

(3 questions)

Lie derivative: Infinitesimal transformation. The notion of Lie derivative. Lie derivative of metric tensor and connection. Motion and affine motion in Riemannian spaces.

Hypersufaces in Euclidean space: Hyperplanes. Hyperspheres. Central quadric hypersurfaces. Reciprocal quadric hypersurfaces. Conjugate radii. Any hypersurface in Euclidean spaces. Riemannian curvature of a hypershpere. Geodesics in a space of positive constant curvature.

(3 questions)

Books Recommended:-

1. C.E. Weatherburn: An Introduction to Riemannian Geometry and the Tensor Calculus, Cambridge University Press, 1966.
2. K. Yano: The Theory of Lie Derivatives an its Applications, North Holland Publishing Company, Amsterdam,1957.
3. R. S. Mishra: A Course in Tensors with Applications to Riemannian Geometry, Pothishala (Pvt.) Ltd., 1965.

Paper IV (b) General Relativity and Gravitation

Basics of Tensor, Tensor density, Levi-Civita tensor, Riemannian metric, Parallel transport and Geodesic, Geodesic Deviation. Riemannian curvature tensor, Parallel propagation and Riemannian curvature tensor, Ricci tensor, Bianchi identities, Conformal curvature tensor, Conformal Invariance, Exterior Derivatives, Lie derivatives. Minkowski Space time, Curved Space time.

(2 questions)

Introduction to General Relativity, Principal of Equivalence, Principal of General covariance, Mach's Principle, Non-Euclidean character of rotating disc, geodesic postulate, Newtonian approximation of equation of motion, Search for Einstein's field equation, Gravitational field in empty space, Clock Paradox, Schwarzschild exterior solution, Singularities in Schwarzschild line element, Isotropic form of Schwarzschild exterior line element, Planetary orbits, Three Crucial tests in General Relativity, Radar Echo Delay (Fourth test), Analogous to Kepler's Law, Energy momentum tensor, Formula for energy momentum tensor for perfect fluid, Schwarzschild internal solution, Boundary conditions, Action Principle, Derivation of Einstein's field equation from variational principle, Energy momentum pseudo tensor, Birkhoff's theorem, Reissner- Nordstrom Solution ,Vaidya metric, The Vaidya metric in Null Co-ordinates.

(4 Questions)

Books Recommended:-

1. K. D. Krori : Fundamentals of Special and General Relativity; PHI Publication, 2010.
2. S. R. Roy and Raj Bali : Theory of Relativity; Jaipur Publishing House, 2008.
3. Steven Weinberg : Gravitation and Cosmology : Principles and applications of General Relativity; Wiley Publ.,2005.
4. J. V. Narlikar : An Introductions to Relativity; Cambridge University Press, 2010.
5. I.B. Khriplovich : General Relativity; Springer Science + business media, 2005.

Paper IV (c) Complex Manifolds

Almost Complex Manifolds : Elementary notions, Nijenhuis tensor Eigen values of F, Integrability conditions, Contravariant and covariant analytic vectors, F-connection.

Almost Hermit Manifolds: Definition, Almost analytic vector fields. Curvature tensor. Linear connections.

(3 questions)

Kaehler Manifolds: Definition. Curvature tensor. Affine connection. Properties of projective, conformal, concircular and conharmonic curvature tensors. Contravariant almost analytic vector. Nearly Kaehler Manifolds: Introduction, Curvature identities, Almost analytic vectors.

(3 questions)

Books Recommended:-

1. R.S. Mishra: Structure on differentiable manifold and their application, Chandrama Prakashan, Allahabad, 1984.
2. K. Yano and M. Kon: Structures of Manifolds, World Scientific Publishing Co. Pvt. Ltd., 1984.

Paper IV (d) Advanced Topology

Separated sets. Connectedness in terms of separated sets. Characterization of connected sets in terms of open sets and closed sets. Closure of a connected set. Union of connected sets. Connected sets in R. Continuity of a function and connectedness. Components and partition of space.

Inadequacy of sequential convergence. Directed sets, nets and subnets and their examples Convergence of a net, charaterisation of open sets, closed sets, closure, cluster point and limit point of a set in terms of net convergence. Hausdorffness and continuity of a function in terms of nets.

(2 questions)

Definition of filter and its examples. Neighborhood filter. Comparison of filters. Filter base and subbase. Convergence of a filter. Ultrafilters. Continuous functions and filters.Net based on filter and filter based on net.

Quotient topology, quotient space, quotient map, quotient space X/R, Finite product space, projection mapping.

Tychonoff product topology in terms standard subbase and its characterizations in terms of projection maps, continuous functions, Product of T_0,T_1,T_2, spaces. Connectedness and compactness, first and second countability for product spaces.

(3 questions)

Books Recommended:-

1. George F. Simmons : Introduction to Topology and Modern Analysis, Mc Graw-Hill Book Company 1963.
2. J.L. Kelley : General Topology, Van Nostrand, Reinhold Co., New York 1995.
3. K.D. Joshi : Introduction to General Topology, Wiley Eastern Ltd., 1983.
4. James R Munkres : Topology, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
5. S. Willard : General Topology Addison-Wesley, Reading, 1970.

Paper V (a) Discrete Mathematics

Semigroups & Monoids: Definition and examples of Semigroups and Monoids. Homomorphism of Semigroups and Monoids. Congruence relation and Quotient Semigroups. Subsemigroup and Submonoids. Direct products. Basic homomorphism theorem.

Lattices: Lattices as partially ordered sets. Their properties. Lattices as Algebraic Systems. Sublattices. Direct products and Homomorphisms. Some Special Lattices e.g., Complete, Complemented and Distributive Lattices.

(3 questions)

Boolean Algebras: Boolean Algebras as Lattices, Various Boolean Identities. The Switching Algebra example. Subalgebras. Direct Products and Homomorphisms. Join-irreducible elements, Atoms and Minterms. Boolean Forms and their Euivalence.

Graph Theory: Definition of Graphs, Paths, Circuits, Cycles & Subgraphs. Induced Subgraphs. Degree of a vertex. Connectivity. Planar graphs and their properties. Trees. Euler's Formula for connected planar graphs.

(3 questions)

Books Recommended:-

1. C.L. Liu: Elements of Discrete Mathematics (Second Edition), McGraw Hill, International Edition, Computer Science Series, 1986.
2. J.P. Tremblay & R. Manohar: Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.
3. N. Dew. Graph Theory with Application to Engineering and Computer Sciences, Prentice Hall of India.

Paper V (b) Mathematical Modeling

Mathematical Modelling: Need, techniques, classification and simple illustrations of mathematical modelling. Limitations of mathematical modelling.

Mathematical Modelling Through Ordinary Differential Equations of First Order:

Linear and Non-linear Growth and Decay models. Compartment models. Mathematical modelling of geometrical problems through ordinary differential equations of first order.

(3 questions)

Mathematical Modelling Through System of Ordinary Differential Equations of First Order: Mathematical modelling in Population Dynamics. Mathematical modelling of epidemics. Compartment models. Mathematical modelling in Economics. Mathematical models in Medicine.

Mathematical Modelling Through Ordinary Differential Equations of Second Order: Mathematical modelling through linear differential equations of second order, Application of Differential Equation in Cardiography.

Mathematical Modelling Through Partial Differential Equations: Situations giving rise to partial differential equation models. Application of Partial Differential Equation in Nuclear Reactors.

(3 questions)

Books Recommended:-

1. J.N. Kapur: Mathematical Modelling, New Age International (P) Limited, New Delhi.
2. Zafar Ahsan : Differential Equations and Their Applications, PH I learning Private Limited, New Delhi.

Paper V (c) Numerical Solution of Differential Equations

Numerical Solution of parabolic partial differential equations (PDE) in one space: two and three levels explicit and implicit differences schemes. Convergence and stability analysis.

Numerical solution parabolic PDE of second order on two spaces dimension: Implicit methods, alternating direction implicit (ADI) methods. Non-linear initial BVP (boundary valued problems).

Differences schemes for parabolic PDE in spherical and cylindrical coordinate systems in one dimension.

(3 questions)

Numerical solution of hyperbolic PDE in one and two spaces dimension: explicit and implicit schemes: ADI methods. Differences schemes for first order equations.

Numerical solutions of elliptical equations, approximations of Laplace and bi-harmonic operators. Solutions of Dirichlet, Neuman and mixed type problems.

Finite element methods: Linear, triangular elements and rectangular elements.

(3 questions)

Recommended Books:

1. M. K. Jain, S.R.K. Iyenger and R. K. Jain: Computational Methods for Partial differential equations, Wiley Eastern, 1994.
2. M. K. Jain, Numerical Solution Differential Equation, 2nd Edition, Wiley Eastern.
3. S. S. Sastry, Introductory Methods of Numerical analysis, Prentice-Hall of India, 2002.
4. D .V. Griffiths and I. M. Smith, Numerical Methods of Engineers, Oxford University Press, 1003.
5. C. F. General and P.O. Wheatley: Applied Numerical Analysis, Addison-Wiley, 1998.

Paper V (d) Algorithm and Data Structure

Fundamentals of *C* Programming, Structures, Pointers. Introduction to the concepts of an abstract data structures and its implementation.

Mathematical Basis: Asymptotic notation, Summations, Recursion formulas.

(3 questions)

Basic data structures: Stacks, queues, lists, tress, priority queues, tables.

Searching Methods: Binary search Tree.

Sorting: General Background, Insertion sorts, Merge sorts and Heap sort.

(3 questions)

Recommended Books:

1 Y. Langsam, M.J. Augenstein, A. M. Tanenbaum: Data Structures using *C* and C^{++}, PHI, New Delhi, 2002.

2 T. H. Cormen, C. E. Leiserson, R. C. Rivest: Algorithms, PHI New Delhi, 2001.

3 B.W. Kernighan, D. M. Ritche: The C Programming Language, 2nd Edition, Prentice Hall, 1989.

M.A./M.Sc. Semester-IV

Paper I Measure Theory

Lebesgue's outer measure λ and its properties. Length of an interval and Lebesgue outer measure. Lebesgue measurable sets in R and σ-algebra of Lebesgue measurable sets M_λ in R Lebesgue measurability of open sets, closed sets and Borel sets. Labesgue measure on R. Example of a Non-Lebesgue measurable set. Cantor's set and its Labesgue measure.

General outer measure μ. Caratheodory's definition of μ-measurable sets. σ-algebra of μ-measurable sets M_μ. Definition of a measure. Measurable space and a measure space.

(2 questions)

Definition of a measurable function. Equivalent conditions for measurable function. Sum and product of measurable functions. Composition of a measurable and a continuous function. Sequences of measurable functions. Measurability of supremum function, infimun function, limit superior function, limit inferior function and limit function. Simple measurable functions and their properties. A non-negative measurable function as the limit of a sequence of non-negative simple measurable functions. Concept of almost everywhere (a.e.). Lebesgue theorem.

Convergence in Measure and its properties. F. Riesz theorem and Egorov theorem. Convergence almost everywhere, almost uniform convergence and their inter-relations. Lebesgue Integration of a simple measurable function on R and its properties. Lebesgue Integration of a bounded measurable function on a set E with finite Lebesgue measure, i.e. $\lambda(E) < \infty$, and its properties. Bounded convergence theorem, Lebesgue integration and Riemann integration. Integration on a measure space. Integration of a non-negative measurable function on a measure space. Lebesgue integral of general measurable function and its properties. Space of Lebesgue integrable functions.

(4 questions)

Books Recommended:-

1. Walter Rudin: Principles of Mathematical Analysis (3rd edition), McGraw-Hill, Kogakusha, 1976 International Student Edition.
2. H. L. Royden : Real Analysis, Macmillan Pub. Co. Inc. New York, 4th Edition, 1993.
3. Richard Johnson Baugh; Foundation of Mathematical Analysis.
4. G. de Barra : Measure theory and Integration, Wiley Eastern Limited, 1981.
5. E. Hewitt & K. Strumberg: Real and Abstract Analysis, Springer – Verlag, New York, 1969.

Paper II - Hilbert Spaces

Inner product spaces, their basic properties and examples, Schwartz inequality. Norm induced by inner product, Continuity of inner product, Hilbert spaces and their examples. Parallelogram equality, polarization identity. Characterization of inner product in terms of norm. Separable Hilbert spaces and their examples

Orthogonal vectors. Orthogonal complement. Projection theorem. Projection operators. Orthogonal sets and their advantage over its linearly independent sets. Complete orthonormal sets.

(3 questions)

Bessel's generalized inequality. Parseval's Relation. Grahm-Schmidt orthogonalization process. Fourier series representation

Bounded linear functionals on Hilbert spaces. Riesz-Frechet representation theorem.Dual spaces.Inner product structure of dual spaces. Reflexivity of Hilbert spaces.

Hilbert adjoint operators. Shift operators. Special cases of Hilbert adjoint operoters self adjoint operators, positive operator, normal operators, unitary operators. Orthogonal projection operators.

(3 questions)

Books Recommended:-

1. P.K. Jain, O.P. Ahuja and K. Ahmad: Functional Analysis, New Age International (P) Ltd. and Wiley Eastern Ltd., New Delhi, 1997.
2. B. Choudhary.& S. Nanda: Functional Analysis with Applications, Wiley Eastern Ltd., 1989.
3. I.J Maddox: Functional Analysis, Cambridge University Press (1970).
4. G.F. Simmons: Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, New York, 1963.
5. K. Chandrashekhara Rao. Functional Analysis, Narosa Publishing House, New Delhi

Paper III- Analytical Dynamics

Classification of dynamical systems. Generalized coordinates. Holonomic and non-holonomic systems, Kinetic energy. Generalized components of momentum. Generalized Components of the effective and applied forces. Lagrange's equations. Examples, Energy equation from Lagrange's equation. Reciprocal relations. Lagrange's equation for impulsive motion. Ignoration of coordinates. The Routhian functin. Euler's equation from Lagrange's equation.

(3 questions)

Hamilton's equations of motion. Application of Hamiltonian methods. Natural motions. The space of events. Action. Hamilton's principle. Principle of least action. Hamilton-Jocobi equation. Hamilton characteristic function. Generating function. Canonical transformations. Phase space. Bilinear invariants. Poisson brackets, Lagrange brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations. Small oscillations. Langrange's equation of small oscillations. Lagrange's determinants. Normal modes and normal coordinates and their stationary properties.

(3 questions)

Books Recommended:-

1. S.L. Loney: An Elementary Treatise on the Dynamics of a Particle and a Rigid Body, Macmillan India Ltd., 1982.
2. A.S. Ramsey: Dynamics part-II, The English Language Book Society and Cambridge University Press, 1972.
3. .J.L. Synge and B.A. Griffith: Principles of Mechanics, McGraw Hill International Book Company, 1982

Paper IV (a)- Finsler Geometry

Finsler metric function. Its properties. Tangent space. Indicatrix. Metric tensor and C-tensor Homogeneity properties of g_{ij} and C_{ijk} . Dual tangent space. Geodesics. δ-differentiation. Partial δ-differentiation. Properties of partial δ-differentiation.

(3 questions)

Fundamental postulates of Cartan. Cartan's covariant derivatives and their properties. Geometry of paths. Berwald's covariant derivative and its properties.

Commutation formula resulting from partial δ-differentiation. Other commutation formulae. Three curvature tensors of Cartan. Identities satisfied by curvature tensors including Bianchi identities.

(3 questions)

Books Recommended:-

1. H. Rund: The Differential Geometry of Finsler Spaces, Springer-Verlag, 1959.
2. M. Matsumoto: Foundations of Finsler Geometry and special Finsler spaces, Kaiseisha Press, Saikawa, Otsu, 520 Japan, 1986.

Paper IV (b)- Cosmology

Static cosmological models, Properties of Einstein Universe, Properties of de-Sitter Universe, Non-Static cosmological models, Friedmann-Robertson-Walker (FRW) cosmological models, Geometrical features of FRW metric, Observable parameters in FRW metric, Particles Horizon, Event Horizon, Einstein's field equation and dynamics of the universe, Cosmologies with a non zero Λ.

(3 questions)

Origin and Evolution of Universe, Creation of matter, C Field Theory, Cosmological equations, explosive Creation, The large number hypothesis, Observable parameters of the Steady State Theory.

Gravitational Collapse, Gravitational Collapse of a Homogeneous Dust ball, Black Holes (Strong Gravitational fields), The Kerr metric or the Rotating black Holes, Ker-Newmann metric.

(3 questions)

Books Recommended:-

1. K. D. Krori : Fundamentals of Special and General Relativity; PHI Publication, 2010.
2. S. R. Roy and Raj Bali : Theory of Relativity; Jaipur Publishing House, 2008.
3. Steven Weinberg : Gravitation and Cosmology : Principles and applications of General Relativity; Wiley Publ.,2005.
4. J. V. Narlikar : An Introductions to Relativity; Cambridge University Press, 2010.
5. I.B. Khriplovich : General Relativity; Springer Science + business media, 2005.

Paper IV (c)- Contact Manifolds

Almost Contact Manifolds : Definition. Eigen values of F. Intergrability conditions of $\pi_{m,}$ π_m and π_1 . Lie derivative. Normal contact structure. Affinily almost cosymplectic manifold.

(3 questions)

Almost Grayan Manifolds: Introduction. D-conformal transformation. Particular affine connections. Almost Sasakian manifold. Quasi-Sasakian manifold.
Sasakian Manifolds: K-contact Riemannian manifold and its properties, Sasakian manifold and its properties. Properties of projective, conformal, concircular and con- harmonic curvatures in Sasakian manifold. Cosymplectic structure.

(3 questions)

Books Recommended:-

1. R.S. Mishra: Structures on differentiable manifold and their applications, Chandrama Prakashan, Allahabad, 1984.
2. K. Yano and M. Kon: Structure of Manifolds, World Scientific Publishing Co. Pvt. Ltd., 1984.

Paper IV (d)- Wavelet Analysis

Wavelet Transform and its Basic Properties : Introduction. Continuous wavelet transform and examples. Basic properties of wavelet transform. Discrete wavelet transform. Orthonormal wavelets.

(3 questions)

Different Ways of Constructing Wavelets: Orthonormal bases generated by a single function. Balaon-Low theorem. Smooth projections on $L^2(R)$. Local sine and cosine bases and the construction of some wavelets.

(3 questions)

Books Recommended:-

1. Eugenio Hernandez and Guido Weiss: A First Course of Wavelets, CRC Press, New York, 1996.
2. C.K. Chui: An Introduction to Wavelets, Academic Press,1992.
3. I. Daubechies: Ten Lectures on Wavelets, CB5-NSF Regional Conference in Applied Mathematics, 61, SIAM 1992.
4. Y. Mayer: Wavelets, algorithms and applications (Translated by R.D. Rayan, SIAM, 1993).
5. M. V. Wikerhauser: Adopted wavelet analysis from theory of software, Wellesley, MA, A,K. Peters, 1994.

Paper V (a)- Information Theory

Measure of Information-Axiom for a measure of uncertainty. The Shannon entropy and its properties. Joint and conditional entropies. Transformation and its properties.

Noiseless coding- Ingredients of noiseless coding problem. Uniquely decipherable codes. Necessary and sufficient condition for the existence of instantaneous codes.Construction of optimal codes.

Discrete Memoryless Channel-Classification of channels. Information processed by a channel. Calculation of channel capacity. Decoding schemes. The ideal observer .The fundamental theorem of information theory and its strong and weak converses.

Continuous Channels-The time-discrete Gaussian channel. Uncertainty of an absolutely continuous variable. The converse of the coding theorem for time-discrete Gassian channel. The time-continuous Gaussian channel. Band-limited channels.

(3 questions)

Some imuitive properties of a measure of entropy-Symmetric,normalization,expansibility,boundedness,recursivity,maximality,stability,additivity,subadditivity,nonnegativity,continuity,branching etc. and interconnections among them. Axiomatic charachterization of the Shannon entropy due to Shannon and Fadeev.

Information functions, the fundamental equation of information, information functions continuous at the origin, nonnegative bounded information functions, measurable information functionsand entropy. Axiomatic charachterisations of the Shannon entropy due to Tverberg and Leo. The general solution of fundamental equation of equation of information. Derivations and their role in the study of information functions.

The branching property.Some charechterizations of the Shannon entropy based upon the branching property. Entropies with sum property.The Shannon inequality.Subadditive, additive entropies.

The Renji entropies. Entropies and mean values. Average entropies and their equality, optimal coding and the Renji entropies.Charechterization of some measures of average code length.

(3 questions)

Books Recommended:-

1. R. Ash, Information Theory, Interscience Publishers, New York, 1965.
2. F.M. Reza, An introduction to Information Theory, MacGraw-Hill Book Company Inc., 1961.
3. J. Aczel and Z .Daroczy, On measures of information and their charechterizations, Academic Press, New York.

Paper V (b)- Bio Mathematics

Introduction, Definition and Scope of Bio-Mathematics, Role of Mathematics in Biosciences. Basic concepts of Fluid Dynamics, Bio-Fluid Dynamics, Basic concepts about blood, Cardiovascular system and blood flows, Blood flow through artery with mild stenosis, Two-layered flow in a tube with mild stenosis, Pulsatile Flow of Blood. Peristaltic flow in tubes and channels.

(3 questions)

Human Respiratory System, Gas exchange and air flow in human lungs. Consumption and transport of Oxygen. Weibel's model for flows in human lung airways. Comparison between flows of blood and flows in lung airways. Diffusion. Fick's laws of diffusion. Diffusion equation and its solution. Modification of the diffusion equation. Diffusion in artificial kidney. Hemodialyser. Types of Hemodialyser.

(3 questions)

Books Recommended:-

1. J.N. Kapur: Mathematical Models in Biology and Medicine, Affiliated East-West Press Pvt. Ltd., New Delhi, 1985.
2. Y.C. Fung: Bio-Mechanics, Springer-Verlag New York Inc., 1990.
3. Stanley E. Charm and George S. Kurland: Blood Flow and Microcirculation, John Wiley & Sons, 1974.
4. S.A. Levin: Frontiers in Mathematical Biology, Springer-Verlag, 1994.
5. S.K. Pundir & R. Pundir : Biomathematics, Pragati Prakashan, 2010.

Paper V (c)- Magneto Hydrodynamics

Maxwell equations. Electromagnetic field in a conductor. MHD approximations. Rate of flow of charge. Important MHD parameters. Diffusion of magnetic field. Frozen-in-fields. Integral of magnetic field equation. Analogy of magnetic field with vorticity. Alfven theorem. Lorentz force and its transformations. Magnetic energy. Poynting vector theorems. Basic equations of in viscid and viscous magnetohydrodynamics. Energy conservation law.

(3 questions)

Alfven waves. MHD waves in a compressible fluid. Equi-partition of energy of Alfven waves. MHD boundary conditions. Equations of incompressible MHD flow. Parallel steady flow. Steady parallel flow in a conservative field of force. One-dimensional steady viscous MHD flow. Hartmann flow. Couette flow.

(3 questions)

Books Recommended:-

1. Alan Jeffery, Magnetohydrodynamics, Oliver and Boyd Ltd., Edinburgh, 1966.
2. F. Chorlton, Text Book on Fluid Dynamics, C.B.S. Publishers, Delhi,1985.
3. S.I. Pai, Magnetohydrodynamics and Plasma Dynamics, Springer-Verlag,1962.

Paper V (d)- Computational Mathematics

Basics of LATEX, Basics of *SciLab* Programming, normal (composite) expression, symbols and variables, Dynamic data typing, logical operators, Loops, Examples of functions of Single arguments, Functions of several variables, Functions with multiple definitions, Functions on lists and functional programming,

(3 Questions)

Nest Family, Operators on functions, Writing efficient programs: some techniques and applications, Solving Physical problems: Differential equations, basics of data analysis with *SciLab*, Graphics, 2D, 3D Graphics.

(3 Questions)

Recommended Books:-

1. *SCILAB (A Free Software To MATLAB),* Achuthsankar S Nair, S. Chand Publishing, 2012.
2. *Scilab: A Practical Introduction to Programming and Problem Solving*, Tejas B. Sheth , CreateSpace Independent Publishing Platform, 2016.
3. *Modeling and Simulation in Scilab/Scicos with ScicosLab 4.4,* Campbell**,** Stephen L., Chancelier**,** Jean-Philippe**,** Nikoukhah**,** Ramine**,** Springer, 2010.

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