Statistical Physics: Ensembles

Ensembles

As a system is defined by the collection of a large number of particles, so the “ensembles” can be defined as a collection of a number of **macroscopically identical** but **essentially independent systems**.

Here the term macroscopically identical means that each of the systems constituting an ensemble satisfies the same macroscopic conditions, like Volume, Energy, Pressure, Temperature and the total number of particles etc.

Here again, the term essentially independent means the system (in the ensemble) is mutually non-interacting to others, i.e., the systems differ in microscopic conditions like parity, symmetry, quantum states etc.

Types of Ensembles

There are three types of ensembles:

1. Micro-canonical Ensemble
2. Canonical Ensemble
3. Grand Canonical Ensemble

Micro-canonical Ensemble

It is the collection of a large number of essentially independent systems having the **same energy E, volume V** and **total number of particles N.**

The systems of a micro-canonical ensemble are separated by rigid impermeable and insulated walls, such that the values of **E, V & N** are not affected by the mutual pressure of other systems.

This ensemble is as shown in the figure below.

Here all the borders are impermeable and insulated.

System 1 Energy E Volume V Number of Particles N	System 2 Energy E Volume V Number of Particles N	System 3 Energy E Volume V Number of Particles N	System 4 Energy E Volume V Number of Particles N	System 5 Energy E Volume V Number of Particles N
System 6 Energy E Volume V Number of Particles N	System 7 Energy E Volume V Number of Particles N	System 8 Energy E Volume V Number of Particles N	System 9 Energy E Volume V Number of Particles N	System 10 Energy E Volume V Number of Particles N
System 11 Energy E Volume V Number of Particles N	System 12 Energy E Volume V Number of Particles N	System 13 Energy E Volume V Number of Particles N	System 14 Energy E Volume V Number of Particles N	System 15 Energy E Volume V Number of Particles N
System 16 Energy E Volume V Number of Particles N	System 17 Energy E Volume V Number of Particles N	System 18 Energy E Volume V Number of Particles N	System 19 Energy E Volume V Number of Particles N	System 20 Energy E Volume V Number of Particles N
System 21 Energy E Volume V Number of Particles N	System 22 Energy E Volume V Number of Particles N	System 23 Energy E Volume V Number of Particles N	System 24 Energy E Volume V Number of Particles N	System 25 Energy E Volume V Number of Particles N
System 26 Energy E Volume V Number of Particles N	System 27 Energy E Volume V Number of Particles N	System 28 Energy E Volume V Number of Particles N	System 29 Energy E Volume V Number of Particles N	System 30 Energy E Volume V Number of Particles N

All the walls here are rigid, impermeable and insulated.

Canonical Ensemble

It's the collection of a large number of essentially independent systems having the same **temperature T, volume V** and **the number of particles N**.

The equality of temperature of all the systems can be achieved by bringing all the systems in thermal contact. Hence, in this ensemble, the systems are separated by rigid, impermeable but **conducting** walls, the outer walls of the ensemble are perfectly insulated and impermeable though.

This ensemble is as shown in the figure:


System 1 Temp. T Volume V Number of Particles N
System 2 Temp. T Volume V Number of Particles N
System 3 Temp. T Volume V Number of Particles N
System 4 Temp. T Volume V Number of Particles N
System 5 Temp. T Volume V Number of Particles N
System 6 Temp. T Volume V Number of Particles N
System 7 Temp. T Volume V Number of Particles N
System 8 Temp. T Volume V Number of Particles N
System 9 Temp. T Volume V Number of Particles N
System 10 Temp. T Volume V Number of Particles N
System 11 Temp. T Volume V Number of Particles N
System 12 Temp. T Volume V Number of Particles N
System 13 Temp. T Volume V Number of Particles N
System 14 Temp. T Volume V Number of Particles N
System 15 Temp. T Volume V Number of Particles N
System 16 Temp. T Volume V Number of Particles N
System 17 Temp. T Volume V Number of Particles N
System 18 Temp. T Volume V Number of Particles N
System 19 Temp. T Volume V Number of Particles N
System 20 Temp. T Volume V Number of Particles N
System 21 Temp. T Volume V Number of Particles N
System 22 Temp. T Volume V Number of Particles N
System 23 Temp. T Volume V Number of Particles N
System 24 Temp. T Volume V Number of Particles N
System 25 Temp. T Volume V Number of Particles N
System 26 Temp. T Volume V Number of Particles N
System 27 Temp. T Volume V Number of Particles N
System 28 Temp. T Volume V Number of Particles N
System 29 Temp. T Volume V Number of Particles N
System 30 Temp. T Volume V Number of Particles N
Outer walls here are rigid, impermeable and insulated.
Inner walls are rigid, impermeable but conducting.

Here, the borders in bold shade are both insulated and impermeable, while the borders in light shade are conducting and impermeable.

Grand Canonical Ensemble

It is the collection of a large number of essentially independent systems having the same **temperature T, volume V & chemical potential μ.**

The systems of a grand canonical ensemble are separated by rigid permeable and conducting walls. This ensemble is as shown in the figure:


System 1 Temp. T Volume V Chemical Potential μ
System 2 Temp. T Volume V Chemical Potential μ
System 3 Temp. T Volume V Chemical Potential μ
System 4 Temp. T Volume V Chemical Potential μ
System 5 Temp. T Volume V Chemical Potential μ
System 6 Temp. T Volume V Chemical Potential μ
System 7 Temp. T Volume V Chemical Potential μ
System 8 Temp. T Volume V Chemical Potential μ
System 9 Temp. T Volume V Chemical Potential μ
System 10 Temp. T Volume V Chemical Potential μ
System 11 Temp. T Volume V Chemical Potential μ
System 12 Temp. T Volume V Chemical Potential μ
System 13 Temp. T Volume V Chemical Potential μ
System 14 Temp. T Volume V Chemical Potential μ
System 15 Temp. T Volume V Chemical Potential μ
System 16 Temp. T Volume V Chemical Potential μ
System 17 Temp. T Volume V Chemical Potential μ
System 18 Temp. T Volume V Chemical Potential μ
System 19 Temp. T Volume V Chemical Potential μ
System 20 Temp. T Volume V Chemical Potential μ
System 21 Temp. T Volume V Chemical Potential μ
System 22 Temp. T Volume V Chemical Potential μ
System 23 Temp. T Volume V Chemical Potential μ
System 24 Temp. T Volume V Chemical Potential μ
System 25 Temp. T Volume V Chemical Potential μ
System 26 Temp. T Volume V Chemical Potential μ
System 27 Temp. T Volume V Chemical Potential μ
System 28 Temp. T Volume V Chemical Potential μ
System 29 Temp. T Volume V Chemical Potential μ
System 30 Temp. T Volume V Chemical Potential μ
Outer walls here are rigid, impermeable and insulated.
Inner walls are rigid, permeable and conducting.

Here inner borders are rigid, permeable and conducting, while outer borders are impermeable as well as insulated.

As the inner separating walls are conducting and permeable, the exchange of heat energy as well as that of particles between the system takes place, in such a way that all the systems achieve the same common temperature **T** and chemical potential **μ.**

Ensemble Average

Every statistical quantity has not an exact but an approximate value. The average of a statistical quantity during motion is equal to its ensemble average.

Let $R(x)$ be a statistical quantity along the x-axis and $N(x)$ be the number of phase points in phase space, then **the ensemble average** of the statistical quantity R is defined as,

$$\bar{R} := \frac{\int_{-\infty}^{\infty} R(x)N(x)\mathrm{d}x}{\int_{-\infty}^{\infty} N(x)\mathrm{d}x}$$